

EFFECT OF CONDENSED POWDER PHASE REACTIONS
ON THE STABILITY OF STEADY-STATE COMBUSTION PROCESSES

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A study has been made of the low-frequency stability of powder combustion in a semiclosed chamber, working within the framework of a linear theory with account taken of condensed-phase (k-phase) inertia and evolution of thermal energy. The case treated is that of the first-order reaction. It is shown that k-phase exothermic chemical decomposition increases the stability of the combustion process. The results of numerical computations are interpreted.

1. Formulation of the Problem. Theoretical studies of the stability of powder combustion, in an open or in a semiclosed chamber (see the review of [1], as well as [2], etc.), usually start from the assumption that the k-phase reaction layer is infinitesimally thin and therefore has zero inertia. In actual fact, this layer must have finite dimensions. According to present combustion theories [3], the greater part of the thermal energy required for k-phase combustion comes from reactions occurring in the phase itself.

Let it be supposed that a first-order chemical reaction is taking place in the k-phase. Under steady-state conditions, active component decomposition and thermal energy propagation can be described by the following system of equations

$$\kappa \frac{d^2 T^\circ}{dx^2} - u^\circ \frac{dT^\circ}{dx} + \frac{Qz}{\rho_0 c_0} a^\circ \exp\left(-\frac{E}{R_0 T^\circ}\right) = 0 \quad (1.1)$$

$$(0 > x > -\infty)$$

$$\rho_0 u^\circ \frac{da^\circ}{dx} + za^\circ \exp\left(-\frac{E}{R_0 T^\circ}\right) = 0 \quad (1.2)$$

with the conditions

$$T^\circ(0) = T_1^\circ, T^\circ(-\infty) = T_0, a^\circ(0) = b^\circ, a^\circ(-\infty) = 1,$$

the origin of coordinates being located in the combustion surface.

Here x is a spatial coordinate, $T^\circ(x)$ the temperature, $a^\circ(x)$ the relative reactant concentration, b° the dispersion depth, T_1° the surface temperature, T_0 the initial temperature, u° the combustion rate, ρ_0 , c_0 , and κ the respective density, specific heat capacity, and thermal conductivity, E the activation energy, z the preexponential factor, Q the heat of reaction, and R_0 the molar gas constant, the degree sign indicating the standard state.

The first integral of (1.1) and (1.2) gives a steady-state relation between f° the temperature gradient at the k-phase side of the interface, the dispersion depth, and other factors, namely

$$f^\circ = \frac{u^\circ}{\kappa} (T_1^\circ - T_0) - \frac{Qu^\circ}{\kappa c_0} (1 - b^\circ) \quad (1.3)$$

Let us now solve the system (1.1), (1.2) assuming the combustion rate u° to be known. We first draw on the Arrhenius equation to pass from (1.1), (1.2) to two other equations which, taken together, give a first-approximation description of combustion and thermal energy propagation in the k-phase reaction and Michelson heating layers. The Michelson layer solution can be written as

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$$\theta_0^\circ(\xi) = H \exp(\xi - \xi_1), \quad a_0^\circ(\xi) = 1 \quad (\xi_1 \geq \xi > -\infty) \quad (1.4)$$

Here $\xi, \theta_0^\circ(\xi)$ are, respectively, a dimensionless coordinate and temperature

$$\xi = x u^\circ / \kappa, \quad \theta_0^\circ(\xi) = (T^\circ - T_0) / (T_1^\circ - T_0)$$

the latter expressed relative to the difference $T_1^\circ - T_0$, the subscript zero indicating a quantity measured in the heating layer and the subscript 1 a quantity measured in the reaction layer, ξ_1 is a quantity related to x_1 , the dimensioned depth of reaction layer, by the equation

$$\xi_1 = -x_1 u^\circ / \kappa$$

and H is the dimensionless temperature at the interface between k -phase reaction and heating layers.

Within the reaction layer, the exponential can be approximated by a parabolic function, the coefficients of this function being so chosen that the values of exponential and parabolic functions will be identical on the combustion surface (point $T^\circ = T_1^\circ$) and at points where $\exp[-E/(R_0 T^\circ)]$ is less than $\exp[-E/(R_0 T_1^\circ)]$ by the respective factors $1/e$ and $1/e^2$ [points $T^\circ = ET_1^\circ/(E + R_0 T_1^\circ)$ and $T^\circ = ET_1^\circ/(E + 2R_0 T_1^\circ)$]. Since the rate of steady-state combustion is related to the k -phase kinetic and thermal characteristics by an equation of the form [4] (a similar equation has been independently derived in [5])

$$(u^\circ)^2 = \frac{2\kappa z R_0 (T_1^\circ)^2 \exp[-E/(R_0 T_1^\circ)]}{(1 - b^\circ) \rho c E [T_1^\circ - T_0 - (1 - b^\circ) Q / (2c_0)]} \quad (1.5)$$

(1.1) and (1.2) can be rewritten

$$\frac{d^2 \theta_1^\circ}{d\xi^2} - \frac{d\theta_1^\circ}{d\xi} + \frac{G_1 a_1^\circ}{b^\circ} [1 - B(1 - \theta_1^\circ) + C(1 - \theta_1^\circ)^2] = 0 \quad (1.6)$$

$(0 > \xi \geq \xi_1)$

$$\frac{da_1^\circ}{d\xi} + G_2 a_1^\circ [1 - B(1 - \theta_1^\circ) + C(1 - \theta_1^\circ)^2] = 0 \quad (1.7)$$

with the conditions

$$\theta_1^\circ(0) = 1, \quad a_1^\circ(0) = b^\circ$$

Here $\theta_1^\circ(\xi), a_1^\circ(\xi)$ are the respective dimensionless temperature and reactant concentration in the k -phase reaction layer

$$\lambda = \frac{Q(1 - b^\circ)}{c_0(T_1^\circ - T_0)}, \quad g = \frac{E(T_1^\circ - T_0)}{R_0(T_1^\circ)^2}, \quad B \approx \frac{g(e-1)(3e-1)}{2e^2}$$

$$C \approx \frac{g^2(e-1)^2}{2e^2}, \quad G_1 = \frac{b^\circ g \lambda}{2} \left(1 - \frac{\lambda}{2}\right), \quad G_2 = \frac{(1 - b^\circ)g}{2} \left(1 - \frac{\lambda}{2}\right)$$

The parameter λ represents the ratio of the amount of thermal energy liberated in the k -phase exothermic reaction to the total thermal (physical) energy content of this phase. The reciprocal of g gives a measure of the range of relative temperatures covered by the chemical reaction. Typically, the conditions under which combustion is observed are such that $\lambda \leq 1, g \gg 1$. Thus for nitroglycerine powders at room-temperatures and rocket pressures, one has [3] $\lambda \approx 0.7-0.9, g \approx 10-14$.

Expressing the temperature and concentration by power series in the neighborhood of the point $\xi = 0$, one can pass from (1.6), (1.7) to the following solutions, valid in the k -phase reaction layer

$$\theta_1^\circ = 1 + \alpha \xi + (\alpha - G_1) \xi^2 / 2 + \{\alpha + G_1(G_2 - 1 - \alpha B)\} \xi^3 / 6 + \{\alpha - G_1[1 + 2\alpha(B + \alpha C) - BG_1 + G_2(G_2 - 1 - 3\alpha B)]\} \xi^4 / 24 + \dots \quad (0 \geq \xi \geq \xi_1) \quad (1.8)$$

$$a_1^\circ = b^\circ \{1 - G_2 \xi + G_2(G_2 - \alpha B) \xi^2 / 2 - G_2[2\alpha^2 C + B(\alpha - G_1) - G_2(3\alpha B - G_2)] \xi^3 / 6 + \dots\} \quad (1.9)$$

where $\alpha = d\theta_1^\circ(0)/d\xi$ is a dimensionless temperature gradient on the k -phase side of the combustion surface.

The quantities H and α of (1.4), (1.8), (1.9) are determined from the condition that the temperature and its various derivatives obtained from the two solutions be pairwise identical at the point $\xi = \xi_1$. Expressions for these quantities, valid to within third-order terms, are

$$H = 1 + \xi_1 + (1 + G_1)\xi_1^2/2 + [1 + 2G_1(1 + B - G_2)]\xi_1^3/6$$

$$\alpha = 1 + G_1 \{ \xi_1 + (B - G_2) \xi_1^2/2 + [2C + B(1 - 3G_2 + 2G_1) + G_2^2] \xi_1^3/6 \}$$

Provisionally setting the reaction layer depth equal to the distance from the k-phase surface at which the rate of evolution of thermal energy has fallen to $1/e^2$ of its value on the surface itself, one obtains

$$x_1 = \frac{x}{u^\circ} \left\{ \frac{1 + G_1}{N} - [(M^2 + L^3)^{1/2} - M]^{1/2} + [(M^2 + L^3)^{1/2} + M]^{1/2} \right\}$$

$$N = 1 + 2G_1(1 + B - G_2), \quad L = \frac{2}{N} - \left(\frac{1 + G_1}{N} \right)^2$$

$$M = \left(\frac{1 + G_1}{N} \right)^3 + \frac{3}{N} \left(\frac{2 - \ln b^\circ}{g} - \frac{1 + G_1}{N} \right)$$

Let us now study the stability of steady-state combustion under small perturbations. In a linear approximation, the expressions for the dimensionless pressure, combustion rate, reactant temperatures and concentrations in the k-phase reaction and heating layers, dispersion depth, and surface temperature and temperature gradient, take the form

$$\eta = \frac{p}{p^\circ} = 1 + \eta_1 \psi(\tau), \quad v = \frac{u}{u^\circ} = 1 + v_1 \psi(\tau), \quad \theta_1 = \theta_1^\circ(\xi) + \theta_{11}(\xi) \psi(\tau)$$

$$a_1 = a_1^\circ(\xi) + a_{11}(\xi) \psi(\tau), \quad \theta_0 = \theta_0^\circ(\xi) + \theta_{00}(\xi) \psi(\tau), \quad a_0 = 1$$

$$b = b^\circ + b_1 \psi(\tau), \quad \vartheta = \theta_1(0, \tau) = 1 + \vartheta_1 \psi(\tau),$$

$$\varphi = \frac{\partial \theta_1(0, \tau)}{\partial \xi} = \alpha + \varphi_1 \psi(\tau)$$

where $\tau = t(u^\circ)^2/\kappa$ designating a dimensionless time. The absolute values of $\eta_1, v_1, \theta_{11}(\xi), a_{11}(\xi), \theta_{00}(\xi), b_1, \vartheta_1, \varphi_1$ are much smaller than the corresponding steady-state values, and the function $\psi(\tau)$ gives the time variation of the dimensionless variables.

Following the well-known method of [6], one can now pass from the steady-state equations

$$u^\circ = u^\circ(p, T_0), \quad T_1^\circ = T_1^\circ(p, T_0), \quad b^\circ = b^\circ(p, T_0)$$

to

$$u = u(p, f), \quad T_1 = T_1(p, f), \quad b = b(p, f) \quad (1.10)$$

valid under nonstationary conditions. Here $f = \partial T(0, t)/\partial x$ designates the temperature gradient on the k-phase side of the surface.

Working still in the linear approximation, and combining (1.10) with (1.3), one obtains

$$\left[k(1 - \lambda) + r - 1 + \frac{\lambda m b^\circ}{1 - b^\circ} \right] v_1 = \left[v \left(r - 1 + \frac{\lambda m b^\circ}{1 - b^\circ} \right) - k \left(\mu + \frac{\lambda \beta b^\circ}{1 - b^\circ} \right) \right] \eta_1 + \frac{k(1 - \lambda)}{\alpha} \varphi_1 \quad (1.11)$$

$$\left[k(1 - \lambda) + r - 1 + \frac{\lambda m b^\circ}{1 - b^\circ} \right] \vartheta_1 = \left\{ \mu \left[k(1 - \lambda) - 1 + \frac{\lambda m b^\circ}{1 - b^\circ} \right] - r \left[v(1 - \lambda) + \frac{\lambda \beta b^\circ}{1 - b^\circ} \right] \right\} \eta_1 + \frac{r(1 - \lambda)}{\alpha} \varphi_1 \quad (1.12)$$

$$\left[k(1 - \lambda) + r - 1 + \frac{\lambda m b^\circ}{1 - b^\circ} \right] \frac{b_1}{b^\circ} = \left\{ \beta \left[k(1 - \lambda) + r - 1 \right] - m \left[v(1 - \lambda) + \mu \right] \right\} \eta_1 + \frac{m(1 - \lambda)}{\alpha} \varphi_1 \quad (1.13)$$

The k, v, r, μ, m, β parameters of these equations respectively characterize the dependence of the combustion rate, surface temperature, and dispersion depth on the initial temperature and pressure

$$k = (T_1^\circ - T_0) \left(\frac{\partial \ln u^\circ}{\partial T_0} \right)_p, \quad v = \left(\frac{\partial \ln u^\circ}{\partial \ln p} \right)_{T_0}, \quad r = \left(\frac{\partial T_1^\circ}{\partial T_0} \right)_p$$

$$\mu = \frac{1}{T_1^\circ - T_0} \left(\frac{\partial T_1^\circ}{\partial \ln p} \right)_{T_0}, \quad m = (T_1^\circ - T_0) \left(\frac{\partial \ln b^\circ}{\partial T_0} \right)_p, \quad \beta = \left(\frac{\partial \ln b^\circ}{\partial \ln p} \right)_{T_0}$$

Under nonstationary conditions, the k-phase inertia is expressed through the equations for thermal conduction and chemical kinetics, i.e.,

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} - v \frac{\partial \theta}{\partial \xi} + \frac{Qz\kappa\alpha}{\rho_0 c_0 (T_1^\circ - T_0) (u^\circ)^2} \exp \left[-\frac{E}{R_0 T(\theta)} \right] \quad (1.14)$$

$$(0 > \xi > -\infty)$$

$$\frac{\partial a}{\partial \tau} + v \frac{\partial a}{\partial \xi} + \frac{z\kappa a}{\rho_0 (u^0)^2} \exp\left[-\frac{E}{R_0 T(\theta)}\right] = 0 \quad (1.15)$$

with the conditions

$$\theta(0, \tau) = \vartheta, \quad \theta(-\infty, \tau) = 0, \quad a(0, \tau) = b, \quad a(-\infty, \tau) = 1$$

Proceeding as with the solution of (1.1), (1.2), we now divide the system (1.14), (1.15) into two systems. Linearizing these last two equations over the reaction layer, introducing a parabolic approximation for the exponential, and drawing on (1.5), one is led to

$$\frac{d^2 \theta_{11}}{d\xi^2} - \frac{d\theta_{11}}{d\xi} - \frac{\theta_{11}}{\psi} \frac{d\psi}{d\tau} + \frac{G_1}{b^0} [1 - B(1 - \theta_1^0) + C(1 - \theta_1^0)^2] (a_{11} + g a_1^0 \theta_{11}) - v_1 \frac{d\theta_1^0}{d\xi} = 0 \quad (0 > \xi \geq \xi_1) \quad (1.16)$$

$$\frac{d a_{11}}{d\xi} + G_2 [1 - B(1 - \theta_1^0) + C(1 - \theta_1^0)^2] (a_{11} + g a_1^0 \theta_{11}) + \frac{a_{11}}{\psi} \frac{d\psi}{d\tau} + v_1 \frac{d a_1^0}{d\xi} = 0 \quad (1.17)$$

with the conditions

$$\theta_{11}(0) = \vartheta_1, \quad a_{11}(0) = b_1$$

In the heating layer, linearization of (1.14), (1.15) leads to

$$\frac{d^2 \theta_{00}}{d\xi^2} - \frac{d\theta_{00}}{d\xi} - \frac{\theta_{00}}{\psi} \frac{d\psi}{d\tau} - v_1 \frac{d\theta_0^0}{d\xi} = 0 \quad (1.18)$$

$$a_{00}(\xi) = 0 \quad (\xi_1 \geq \xi > -\infty)$$

with the condition

$$\theta_{00}(-\infty) = 0$$

A first-approximation solution for (1.14), (1.15) is now obtained by integrating (1.16)-(1.18) with a predetermined $\psi(\tau)$ function, adjusting to get temperature distribution coincidence at $\xi = \xi_1$.

The time-variation of the pressure in the semiclosed chamber is given by the mass-balance equation

$$\frac{V}{RT_2} \frac{dp}{dt} = \sigma \rho_0 u - A F_* p \quad (1.19)$$

Here T_2 is the gas temperature in the chamber (assumed constant), V the free volume of the chamber, σ the area of the powder combustion surface, F_* the critical cross-sectional area of the nozzle, R the molar gas constant, and A the escape coefficient. In a linear approximation, (1.19) takes the form

$$\frac{v_1}{\eta_1} = 1 + \frac{\chi}{\psi} \frac{d\psi}{d\tau} \quad (1.20)$$

χ being the ratio of relaxation times for k -phase heating layer (t_1) and chamber (t_2),

$$\chi = \frac{t_2}{t_1}, \quad t_1 = \frac{\kappa}{(u^0)^2}, \quad t_2 = \frac{V}{A F_* R T_2}$$

A stability analysis of the (1.11)-(1.13), (1.16)-(1.18), (1.20) system indicates that continual variation in the damping coefficient over the critical region requires a stable combustion limit satisfying the equations

$$(A_1 k - A_2)(A_3 k - v A_2) - (B_1 k + B_2)(B_3 k - v B_2) = (A_1 k - A_2)^2 + (B_1 k + B_2)^2 \quad (1.21)$$

$$\chi = \frac{(A_1 k - A_2)(B_3 k - v B_2) + (B_1 k + B_2)(A_3 k - v A_2)}{\gamma [(A_1 k - A_2)^2 + (B_1 k + B_2)^2]} \quad (1.22)$$

Here $\gamma = \omega \kappa / (u^0)^2$ is a dimensionless effective frequency and

$$A_1 = (1 - \lambda)(\alpha \Phi_1 - W_1)$$

$$A_2 = (1 - \lambda)(r Y_1 + m X_1) - \alpha \Phi_1 \left(r - 1 + \frac{\lambda m b^0}{1 - b^0} \right)$$

$$A_3 = (1 - \lambda)(\mu Y_1 + \beta X_1) - \alpha \Phi_1 \left(\mu + \frac{\lambda \beta b^0}{1 - b^0} \right)$$

$$\begin{aligned}
B_1 &= (1 - \lambda)(\alpha\Phi_2 + W_2) \\
B_2 &= \alpha\Phi_2 \left(r - 1 + \frac{\lambda mb^0}{1 - b^0} \right) + (1 - \lambda)(rY_2 + mX_2) \\
B_3 &= \alpha\Phi_2 \left(\mu + \frac{\lambda\beta b^0}{1 - b^0} \right) + (1 - \lambda)(\mu Y_2 + \beta X_2) \\
\Phi_1 &= 1 + \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) \left(\xi_1 + \frac{\xi_1^2}{2} + \frac{\xi_1^3}{6} \right) - gG_1 \frac{\xi_1^2}{2} + \left\{ \gamma R_1 + gG_1 \left[3G_2 - 2(B + 1) + \frac{1}{2} \left(1 + \frac{\gamma}{R_1} \right) \right] \right\} \frac{\xi_1^3}{6} \\
\Phi_2 &= R_1 \left[\xi_1 + \frac{\xi_1^2}{2} + (1 - gG_1) \frac{\xi_1^3}{6} \right] - \gamma \left[1 + \left(3 - \frac{\gamma}{R_1} \right) \frac{\xi_1}{6} \right] \frac{\xi_1^2}{2} \\
W_1 &= \frac{HR_1}{\gamma} - \xi_1 - \left(1 - G_2 + (B - G_2)\xi_1 + \frac{1}{2} \left(3 - \frac{\gamma}{R_1} \right) (1 + G_1\xi_1) \right. \\
&\quad \left. - \left[2 + g + B - G_2(2G_2 - 3B) - \frac{1}{2} \left(1 + \frac{\gamma}{R_1} \right) (1 + G_2) \right] \frac{\xi_1}{3} \right) G_1 \frac{\xi_1^2}{2} - \left(2 - \frac{\gamma}{R_1} \right) \frac{\xi_1^3}{6} \\
W_2 &= \frac{H}{2\gamma} \left(1 - \frac{\gamma}{R_1} \right) + R_1 \frac{\xi_1^2}{2} + \{ 2R_1 - \gamma + G_1 [2R_1 - G_2(\gamma + R_1)] \} \frac{\xi_1^3}{6} \\
Y_1 &= \frac{1}{2} \left(1 + \frac{\gamma}{R_1} \right) - \frac{\gamma\xi_1^2}{2} \left[R_1 + (R_1 - \gamma) \frac{\xi_1}{3} \right] + gG_1 \left\{ \xi_1 + \right. \\
&\quad \left. + \left[B(1 + G_1\xi_1) + \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) - 2G_2 \right] \frac{\xi_1^2}{2} + \left[B + 2C - \right. \right. \\
&\quad \left. \left. - (g + B)G_1 + (1 + B) \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) - G_2 \left(1 - \frac{\gamma}{R_1} - 3G_2 + 6B \right) \right] \frac{\xi_1^3}{6} \right\} \\
Y_2 &= R_1 - \gamma\xi_1 - \frac{\gamma\xi_1^3}{4} \left(1 - \frac{\gamma}{R_1} \right) \left(1 + \frac{\xi_1}{3} \right) - gG_1 \frac{\xi_1^2}{2} \left\{ R_1 + [R_1(1 + B - 2G_2) - \gamma(2 + G_2)] \frac{\xi_1}{3} \right\} \\
X_1 &= G_1 \left\{ \xi_1 + \left[B(1 + G_1\xi_1) + \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) - G_2 \right] \frac{\xi_1^2}{2} + \right. \\
&\quad \left. + \left[B + 2C - (g + B)G_1 + \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) (1 + B) - \gamma(\gamma + R_1) - \right. \right. \\
&\quad \left. \left. - G_2 \left(3B + \frac{1}{2} \left(1 - \frac{\gamma}{R_1} \right) - G_2 \right) \right] \frac{\xi_1^3}{6} \right\} \\
X_2 &= -G_1 \frac{\xi_1^2}{2} \left\{ \gamma + R_1 + \left[2\gamma(B - G_2) + R_1(B + 1 - G_2) - \frac{\gamma}{2} \left(1 + \frac{\gamma}{R_1} \right) \right] \frac{\xi_1}{3} \right\} \\
R_1 &= 2^{-1/2} [(1 + 16\gamma^2)^{1/2} - 1]^{1/2}
\end{aligned}$$

The frequency and physicochemical parameters of the k-phase being given, (1.21) and (1.22) can be used to find the values of k and χ at the stability limit.

In the quasi-stationary case ($\gamma \sim 0$), breakdown of stability occurs when

$$\nu = 1 + \frac{2N_1 [1 - \lambda + \mu + \lambda\beta b^0 / (1 - b^0)] - (1 - \lambda)(N_2 + \mu N_3 + \beta N_4)}{\alpha N_1 [r - 1 + \lambda mb^0 / (1 - b^0)] - (1 - \lambda)(rN_2 + mN_4)} k \quad (1.23)$$

where

$$\begin{aligned}
N_1 &= 1 - gG_1 \frac{\xi_1^2}{2} \left[1 + (1 + 2B - 3G_2) \frac{\xi_1}{3} \right] \\
N_2 &= 1 + G_1 \frac{\xi_1^2}{2} \left\{ G_2 + [g + G_2(3B - 2G_2)] \frac{\xi_1}{3} \right\} \\
N_3 &= 1 + gG_1 \left\{ \xi_1 + (B - 2G_2) \frac{\xi_1^2}{2} + [B + 2C + G_1(2B - g) + 3G_2(G_2 - 2B)] \frac{\xi_1^3}{6} \right\} \\
N_4 &= G_1 \left\{ \xi_1 + (B - G_2) \frac{\xi_1^2}{2} + [B + 2C + G_1(2B - g) + G_2(G_2 - 3B)] \frac{\xi_1^3}{6} \right\}
\end{aligned}$$

Calculations show (Table 1) that critical ν values can be either greater or less than unity, under typical combustion conditions. Table 1 shows values of ν_* calculated from (1.23) for various parameter values. Here, and in what follows, the assumption is made that the dispersion depth diminishes with increasing pressure and decreasing initial temperature, relations consistent with estimates based on preliminary experimental data. It is obvious that for the T_S model [7] $\nu_* > 1$, and for Q model $\nu_* < 1$.

TABLE 1

Combustion model	r	μ	λ	g	b°	m	β	ν_*	k_*
[8, 11] [9]	0 $\neq 0$	0 $\neq 0$	0 0	∞				1 1	1
[10] A. D. Margolin			0.84 0.51 0.17		0.5 0.7 0.9				6.4 2.02 1.2
	0	0	0.84		0.5	0.4			4.23
	0	0	0.51		0.7	0.4			1.07
	0	0	0.17		0.9	0.4			0.47
	0	0	0.25	10	0.5	0.2	-0.05	$1-0.04 k$	
	0	0	0.5	10	0.5	0.2	-0.05	$1-0.01 k$	
	0	0	0.75	10	0.5	0.2	-0.05	$1+0.03 k$	
This paper	0	0	0.75	5	0.5	0.2	-0.05	$1-0.01 k$	
	0	0	0.75	15	0.5	0.2	-0.05	$1+0.04 k$	
	0	0	0.75	10	0.5	0	-0.05	$1+0.02 k$	
	0	0	0.75	10	0.5	0.4	-0.05	$1+0.03 k$	
	0	0	0.75	10	0.5	0.2	0	$1-0.02 k$	
	0	0	0.75	10	0.5	0.2	-0.1	$1+0.07 k$	
	1/3	0.1	0.25	10	0.5	0.2	-0.05	$1-0.04 k$	
	1/3	0.1	0.5	10	0.5	0.2	-0.05	$1-0.07 k$	
	1/3	0.1	0.75	10	0.5	0.2	-0.05	$1-0.1 k$	
	1/3	0.1	0.75	5	0.5	0.2	-0.05	$1-0.15 k$	
	1/3	0.1	0.75	15	0.5	0.2	-0.05	$1-0.08 k$	
	1/3	0.1	0.75	10	0.5	0	-0.05	$1-0.08 k$	
	1/3	0.1	0.75	10	0.5	0.4	-0.05	$1-0.13 k$	
	1/3	0.1	0.75	10	0.5	0.2	0	$1-0.16 k$	
	1/3	0.1	0.75	10	0.5	0.2	-0.1	$1-0.03 k$	

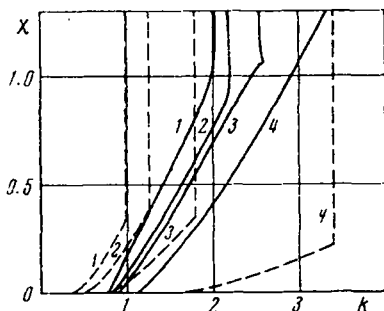


Fig. 1

The condition for breakdown of stability obtained by Ya. B. Zel'dovich [11], namely, $k_* = 1$, follows as the special case of (1.24) with $\lambda \sim 0$.

Values k_* corresponding to various values of the dispersion depth were calculated from (1.24) with $Q = 270$ kcal/mole, $T_1 - T_0 = 400^\circ\text{C}$, $m = 0.4$, and $c_0 = 0.4$ cal/(g·deg) and are shown in Table 1. Critical values of k taken from [10] and [11] are given in this same table for comparison. Taken in conjunction with (1.24), the figures of the table indicate that stable states with $k > 1$ can exist if the k -phase exothermic reaction layer has finite depth.

Equation (1.24) indicates that stable steady-state combustion (at certain values of λ) is possible when $k > 1$, the conditions required being that $\lambda \neq 0$, $b^\circ \ll 1$, and the dispersion low (elevated pressure). Combustion can be unstable, even at $k < 1$, in the other limiting case where the initial temperature is low, $b^\circ \sim 1$, and $m > 0$.

2. Analysis of the Results of Calculations Based on Eqs. (1.21), (1.22). The effect of the λ parameter on the combustion stability is illustrated by Fig. 1. Curves 1, 2, 3, and 4 were developed for $\lambda = 0, 0.25, 0.5, 0.75$, respectively, using the values $\nu = 2/3$, $g = 10$, $b^\circ = 0.5$, $m = 0.2$, $\beta = -0.05$. The dashed-line curves apply to the case of surface temperature constancy ($r = \mu = 0$), the full-line curves to the case of varying surface temperature ($r = 1/3$, $\mu = 0.1$). Similar remarks also apply to Figs. 2, 3, and 4.

Increasing the value of λ considerably extends the region of stable steady-state combustion. Physically, this is to say that the thermal effect of the gaseous phase on the condensed phase diminishes as the

If evolution of thermal energy in the k -phase chemical reaction is neglected ($\lambda \sim 0$), the condition for stability breakdown in the semi-closed chamber, namely $\nu_* = 1$ [8, 9], is again obtained as a special case of (1.23).

Study of the constant-surface-temperature model shows

$$k = \frac{1}{1-\lambda} \left(1 - \frac{\lambda b^\circ m}{1-b^\circ} \right) \quad (\lambda < 1) \quad (1.24)$$

to be a necessary condition for discontinuous loss of combustion stability.

A condition for combustion stability, similar to that of (1.24), was originally obtained by A. D. Margolin [10].

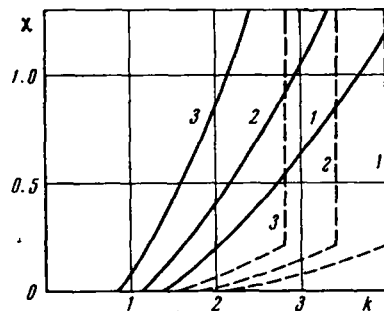


Fig. 2

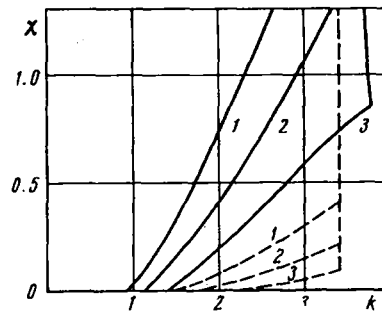


Fig. 3

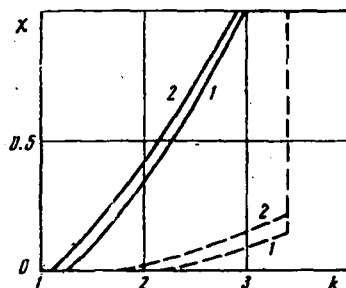


Fig. 4

contribution of the k-phase chemical reaction to the thermal energy content of the powder increases. The thermal state of the k-phase is then more and more nearly determined by the thermal energy liberated in the solid phase reaction, the tendency being toward stabilization of the combustion process.

The fact that the k-phase chemical reaction tends to increase the stability of combustion has already been pointed out in [12, 13], where the problem has been discussed in terms of models, and for operating conditions different from those considered here. The study of [13] started from the improbable assumption that the k-phase reaction is a zeroth-order process. This approach led to an incorrect formulation of the problem and a reduction in the number of characteristic parameters required in a final solution.

Increasing the m parameter has the effect of destabilizing the combustion process (Fig. 2). Curves 1, 2, and 3 of Fig. 2 apply for $m=0, 0.2, 0.4$ for $\nu=2/3, \lambda=0.75, g=10, b^{\circ}=0.5$, and were obtained with the values $\beta=-0.05$. An increase in m implies a reduction in the initial powder temperature, with an accompanying impairment in k-phase heating, the effect of the thermal state of the powder on the gaseous phase becoming more pronounced.

Figure 3 shows limits of combustion stability plotted as a function of the parameter β . Curves 1, 2, and 3 apply for $\beta=0, -0.05, -0.1$. Here it was assumed that $\nu=2/3, \lambda=0.75, g=10, b^{\circ}=0.5, m=0.2$. Reduction in β corresponds to an increase in the pressure.

The relation between the stability limits and the g parameter is shown in Fig. 4. Curves 1 and 2 apply for $g=5, 10$ and were constructed for the case in which $\nu=2/3, \lambda=0.75, b^{\circ}=0.5, m=0.2, \beta=-0.05$. An increase in the g value results in a decrease in the combustion stability; it indicates an increase in the activation energy for the condensed phase reaction, i.e., a contraction of the k-phase zone of thermal energy evolution.

Calculations indicate the mean depth of the reaction layer to be some 20-30% of the value of κ/u° .

The approach adopted here has been phenomenological, account being taken of exothermic k-phase decomposition and dispersion in a generalization of earlier methods.

It has been shown that many experimental results can be correlated in a model in which T_1 is assumed to be constant and account is taken of a possible variation in the dispersion coefficient. This is indication that it is the liberation of thermal energy during chemical reaction, rather than reaction layer inertia, which is the significant factor here.

Comparison of theory and experiment is still a matter of difficulty since there are, as yet, no trustworthy data on the effect of the initial temperature and pressure on the dispersion depth, the one factor which largely fixes the liberation of thermal energy.

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